Summary of 26 Sept ’21 Meeting

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In Sunday’s meeting we covered some questions from Chap. VII, Sects. 1 & 2, including written questions from Randy and Peter. Attendees were Barney, Joe, Perry, Craig and Randy. **Next meeting will be on Oct. 24.** This summary will be posted in Dropbox (Dodson & Poston/Chapter 7) and so can be accessed and modified there (hopefully my version of Word is compatible with everyone; if not, please let me know). Additions, comments and (gasp!) corrections are encouraged.

From Randy’s questions (see *Questions on D&P Chap. VII, Sect. 1.rtf* in Dropbox)

* Ex. VII.1.1b): The exercise asks to find a topology on the set X, which consists of the natural numbers, *i* ∈ℕ , as well as an element representing ∞, which makes the sequence in Def. VI.2.01 (pgs. 125-126) a special case of the definition in part a). I suggested that the discrete topology might work but Joe argued that it would not, citing the fact that any set in the discrete topology is open (by definition of a topology) thus violating the definition of a limit point. In other words, a limit point has an open neighborhood around it containing an infinite number of points (in this case, the natural numbers, *i*, greater than some number n) while only a finite number (*i* < n) lie outside of it. In a discrete topology, these two sets are complementary and thus they both cannot be open.

Joe used the whiteboard to show how Def. VI.2.01 and the definition in part a) are related (he saved it and promises to put it in Dropbox). He also suggested an appropriate topology for this situation, or in his own words

“The topology I came up with was

T = { empty set } union { complement of F | F is a finite subset of (N + ∞) }

It's easy to check that this satisfies the definition of a topology.  Now a neighborhood of ∞ is a set that contains ∞ and excludes finitely many points.”

* Ex. VII.1.2.a-e: It was generally agreed that the main feature of the formidable function f(x,y) presented in this exercise is the parabola y = 2x2 along which the argument of the exponential is zero so that f(x,y) = |x|, which is not differentiable at x = 0. *See also Sam’s plot of the function in his email of 9 Sept (thank you, Sam).* However, approaching zero along the axes (or apparently any straight line), gives well defined partial derivatives (namely zero*).* Details of the analysis in parts c,d & e were glossed over.

Peter’s questions (see Sect. 4.1 of Agenda in Dropbox)

* Understanding space *Lk(T; T’)* of higher derivatives: This refers to the 2nd paragraph of Sect 1.02 (pg. 152). The following is my interpretation (RJP). The 1st line says[[1]](#footnote-1) “Notice that *D*2f = *D*(*D*f) takes values in *L*(*T; L*(*T;T*’))”. This cryptic expression is apparently trying to say that *L*(*T; L*(*T;T*’)) somehow represents the derivative of a derivative.

We know from the previous paragraph that the linear map *D*f = *L*(*T;T’*) = the derivative of *f* at the point ***x*** = (x1,…,xn). *D*f is a very different animal from *f*. It operates on vectors in the domain *T*, emanating from the point ***x***, to give points in the tangent space of *f(x)* (see eg. Fig. 1.1b, pg. 149). For the general case, it must take into account the multi-dimensionality of the domain and image of *f* : ℝn 🡪 ℝm and is thus represented by a matrix (the Jacobian, pg. 154). The components of the matrix are the partial derivatives of *f* with respect to each of the basis vectors in its domain (Sect 1.03, pg. 153).

For higher order derivatives, each component of *D*f represents a map from ***x*** to the value of the partial derivative at f(**x**). Thus the derivative of each component (derivative of a derivative) will also be associated with a matrix itself.

This can be seen more clearly by considering a scalar-valued function *f :* ℝn 🡪 ℝ. As described on pg. 154, the 1st derivative, *D*f, for this case is given by a *1xn* matrix [∂f/∂x1, ∂f/∂x2, …,∂f/∂xn]. The 2nd derivative, not explicitly described in D&P, is given by the *nxn* Hessian[[2]](#footnote-2) matrix {∂2f/∂xi∂xj}, i,j = 1,…,n. For the general case f : ℝn 🡪 ℝm, the 2nd (and higher derivatives) have a more complicated structure better represented by tensor products (pg.152).

* f\* and the “three entries” on pgs. 154-155: At the bottom of pg. 154, they focus on the function f : ℝ 🡪 ℝn which describe a parametric (1-D) curve in ℝn. Using *t* as the parameter instead of *x* in the notation, it would seem to me that f(t) = [f1(t),…,fn(t)] would indeed be a vector. However, others may have better insight as to why D&P say it is not[[3]](#footnote-3). Nonetheless, for f : ℝ 🡪 ℝn we have f\* = Df(t) = [(df1/dt),…, dfn/dt)]

For the record, the three entries on pg. 155 appear to be

1. the linear map Dxf total derivative of f at x ∈ X
2. the number (df/dt)(x) apparently the slope for f : ℝ 🡪 ℝ
3. the vector f\* = [(df1/dt),…, dfn/dt)]

Perhaps they called out these 3 entries to point out different manifestations of *D*f.

* Fig VII.2.4 on pg. 162 showing the different behaviors of the chart φa and φb: The consensus of the group seemed to be that the curve f(ℝ) on the torus has a kink in the area of intersection of Ua and Ub and that φb is able to straighten it out while φa is not. The depiction is pretty lame, though.

The main point seems to be that, per the condition Miii) of Def. 2.01 (pg. 161), the charts must be both Ck on Ua⋂Ub.

1. I’m setting *D* = *D-tilda* here and ignoring the distinction between the derivative in a free vector space *T* and the tangent space *TxX; see pgs. 43 & 151).* [↑](#footnote-ref-1)
2. <https://en.wikipedia.org/wiki/Hessian_matrix> [↑](#footnote-ref-2)
3. I think they are assuming f : ℝ 🡪 ℝ [↑](#footnote-ref-3)